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Secondary flows in ducts of square cross-section

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The paper presents the outcome of experimental research on turbulence-induced secondary flows in square-sectioned ducts. The main emphasis of the experiments has been on the measurement of the secondary flows in a duct with equally roughened surfaces. Here the secondary flow is a substantially larger proportion of the axial flow than is the case in smooth-walled ducts. With the secondary velocities normalized by the friction velocity, however, the resultant profiles for smooth and rough surfaces are the same, within the precision of the measurements.

1. Introduction

More than forty years have elapsed since Nikuradse (1926) discovered that the axial velocity contours in turbulent flow through straight rectangular-sectioned channels bulged towards the corners of the duct. The cause of these bulges was traced by dye injection to the presence of secondary flows in the plane of the duct cross-section which transported high velocity fluid from the centre of the duct towards the corners. Although detection of secondary flows was comparatively easy, their direct measurement was not. Not until Hoagland's (1960) thesis were quantitative profiles of secondary velocity available, and even these data were at variance, in some respects, with the requirements of mass conservation. The difficulty lay in the fact that the secondary velocities were, at most, a few per cent of the primary velocity. This meant that any small distortions of the flow pattern caused by the measuring probe could have an appreciable effect on the secondary velocities that were deduced. The accuracy of Hoagland's data was diminished further by the use of a coarse device for measuring the small angle made by the total velocity vector with the duct axis. Despite the comparative imprecision of the measurements, however, Hoagland's work remains a major contribution.

Although Prandtl (1953) gave some explanation of the origins of these secondary motions, it was not until the work of Brundrett & Baines (1964) that a fairly complete description was provided. They showed that it was gradients in Reynolds stresses in the plane of the cross-section that gave rise to a source of streamwise vorticity. Their work included hot-wire measurements of all six components of the Reynolds stress. From these data they deduced that, in rectangular-sectioned ducts (and with axes chosen parallel to the sides), it was predominantly the normal-stress gradients which generated the velocities in the plane of the cross-section.

Brundrett & Baines, like Gessner & Jones (1965) soon afterwards, employed

Hoagland's hot-wire technique to measure the turbulence-induced mean-flow motions. An interesting difference emerges between these two sets of data: Brundrett & Baines, like Hoagland, found that at any point in the duct the ratio of primary to secondary velocity remained the same when the Reynolds number was varied; Gessner & Jones' measurements indicated, however, that the secondary motion diminished substantially relative to the axial velocity as the Reynolds number was increased by a factor of four. Normalization by the average wall friction velocity (rather than by the axial velocity) substantially diminished this Reynolds number variation.

It was the primary objective of the research described here to resolve whether the axial or friction velocity (if either) provided the appropriate normalizing velocity scale for the secondary flow. However, since the ratio of axial to friction velocity is only a weak function of Reynolds number and the secondary-flow-measurement technique did not provide great precision, measurements would have had to have spanned a wide range of Reynolds numbers to permit a definite conclusion to be drawn. The need for such an extensive programme of measurements could, we felt, be avoided by considering instead flow in a duct with roughened surfaces, because these tests provided a substantially different ratio of friction to bulk velocity than was attainable in a smooth-walled duct. A short description of the apparatus employed is provided in § 2 below and the experimental results are discussed in § 3.

2. Apparatus and instrumentation

Measurements were made in an open-circuit test facility at the Berkeley Nuclear Laboratories of the Central Electricity Generating Board. The principal components and their arrangement are shown schematically in figure 1. Two test sections were employed, each being a square-sectioned duct made from 0·5 in. Perspex sheets and with internal side dimensions of 4 in. The ducts were 23 ft in length and fabricated in six equal sections with flanged ends. The walls of one duct were smooth, while those of the other were roughened by machining in the inner surface square-sectioned ribs 0·030 in. in height pitched 0·214 in. apart.† Air was delivered to the test section by a 10 h.p. centrifugal fan by way of a 20 in.-square settling chamber and a contracting section 6 ft in length. The test sections were instrumented with static pressure taps spaced nominally 11·5 in. apart. For the rough-walled duct, care was taken to ensure that the pressure tappings were symmetrically located with respect to the roughness elements.

The secondary-flow measurements were made in the exit plane of the duct and the axial velocity profiles were measured 2 in. upstream. The probes were held rigidly in a two-way traversing mechanism which permitted the relative co-ordinates of the probes to be determined to within 0·001 in. (the absolute location was fixed by means of a travelling microscope). The technique of measuring the secondary flow has been described in some detail by Brundrett & Baines (1964); here, therefore, a brief report may suffice. A standard Disa hot-wire probe,

† The internal dimension of the duct, 4 in., was measured to the root of the ribs.

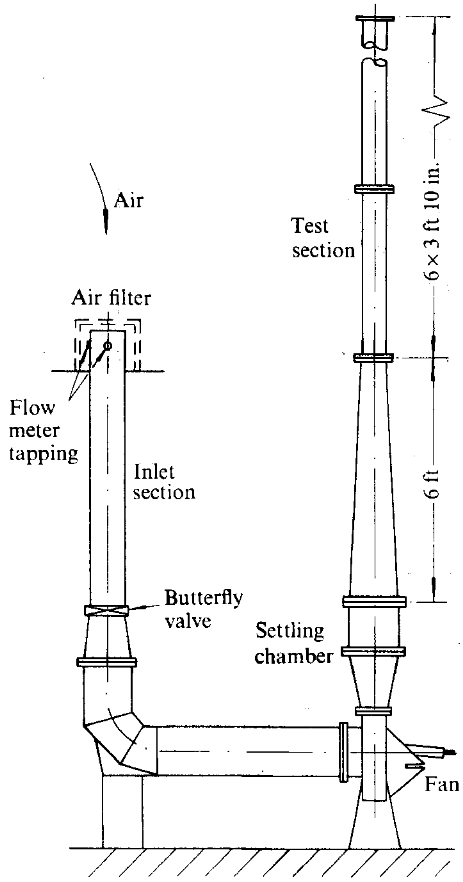

FIGURE 1. Schematic diagram of wind tunnel.

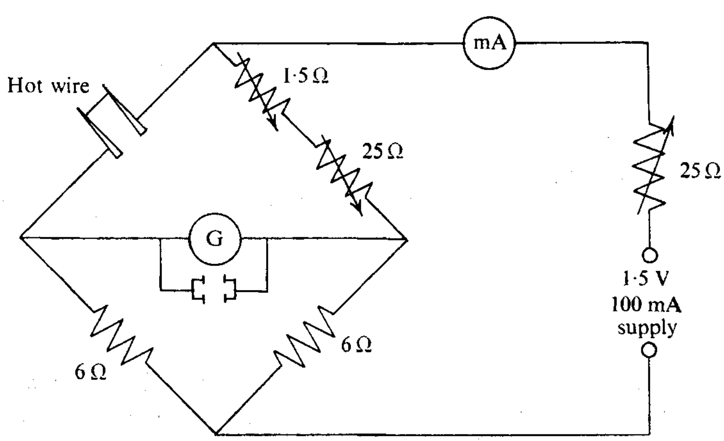

FIGURE 2. Electrical circuit for measurement of secondary flow.

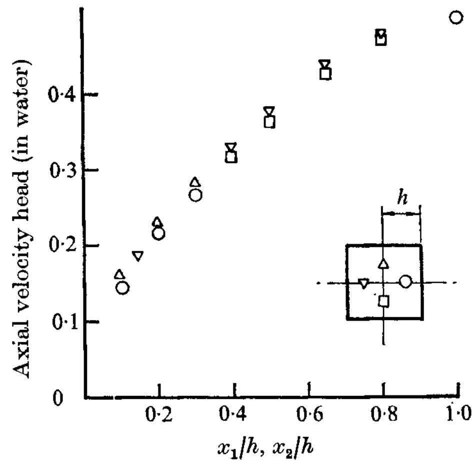

FIGURE 3. Variation of dynamic head along bisectors of sides.

with a single normal wire, was arranged to form one arm of the Wheatstone bridge circuit shown in figure 2. The probe itself was arranged with its axis parallel to one pair of sides and normal to the axis of the duct, with the wire inclined at an angle α to the duct axis. The bridge was then balanced by adjustment of the coarse (25 Ω) and fine (1·5 Ω) resistors. The hot wire was then rotated about its own axis until a position, where the wire made an angle β with the duct axis, was found at which the bridge was again in balance. The bisector of the angle $\alpha+\beta$ gave the true direction of flow. The velocity component in the plane of the cross-section could then be deduced from a knowledge of this flow angle and of the axial velocity (the latter being determined by a Pitot-tube traverse). The deduced direction of flow was, as it should be, effectively independent of the initial inclination α of the wire to the flow. In accord with Hollingsworth's (1967) findings, however, we found that the sensitivity was greatest for initial settings of between 10° and 15°.

A 5 in. radius protractor with a vernier scale enabled the inclination of the wire to be measured to within 0·1°. The accuracy with which the secondary flow angle could be determined was not greater than 0·2°, however, because of uncertainties in aligning the hot wire with the probe axis. In addition, further imprecisions arose through the presence of the probe itself affecting the flow direction, the effect being especially severe when the probe axis lay close to one wall. Fortunately, discrepancies of this kind are readily detectable; the question is discussed further in § 3.

3. Presentation of results

Before the main test programme began, all seams and flanges were checked for leaks and the velocity profile was measured along centre-lines in the exit plane to ascertain the degree of symmetry of the flow. As may be seen from figure 3, the maximum variation in dynamic head at symmetric locations was only about 3 % for the smooth duct; a comparable variation was also measured in the rough

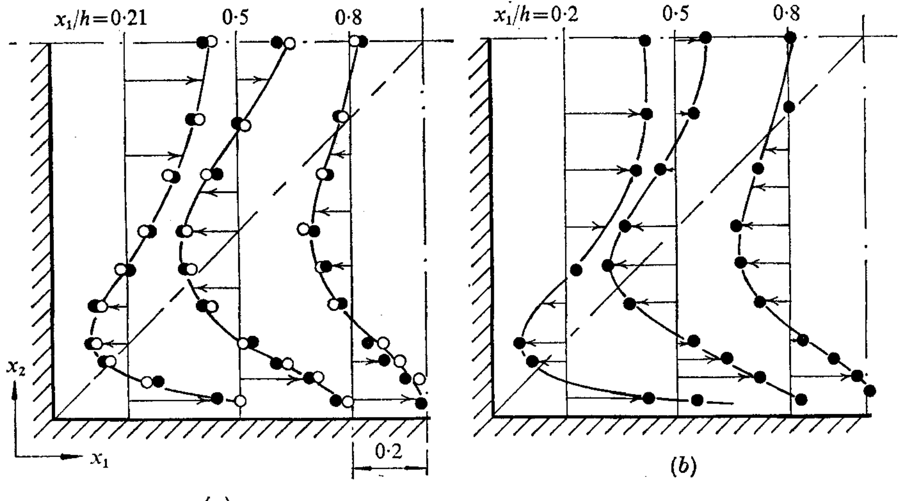

FIGURE 4. Normalized secondary flow profiles U_1/U_τ. ○, $Re = 69000$; ●, $Re = 21500$. (a) Rough duct. (b) Smooth duct.

duct. The principal measurements were made in just one quadrant of the duct. Profiles of axial velocity were obtained at a Reynolds number Re, based on hydraulic diameter, of 21×10^4 for both smooth and rough ducts and at $Re = 6{\cdot}9 \times 10^4$ for the rough duct alone. Secondary flow angles were then measured at the same Reynolds numbers and at the same positions in the duct.

The principal results of the present experiments are contained in figure 4. This shows, for rough- and smooth-walled ducts of side $2h$, profiles of secondary velocity at three positions in the cross-section; in each case the velocity U_1 is normalized by the average wall friction velocity U_τ. It is evident that the secondary flow, directed from the centre towards the corners with return flow to the centre along the bisectors of the sides, is practically the same for the smooth and the rough channels. If, instead, we had used the bulk (or centre-line) axial velocity as the normalizing scale, we should have found that the strength of the secondary flow was about twice as large for the rough channel as for the smooth. These results lend support to Gessner & Jones' (1965) finding that for flow in smooth ducts the secondary velocity normalized with friction velocity was effectively independent of Reynolds number.

A measure of the consistency of the secondary flow data can be obtained by checking how closely the measured profile satisfies the continuity requirement that the flow rate towards the wall should equal that away from it. For both the smooth and the rough ducts the flow rates to and from the wall agree to within about 10 % at $x_1/h = 0{\cdot}8$ and $0{\cdot}5$. In view of the inherent imprecision in obtaining the data, this level of agreement may be regarded as satisfactory. At the position closest to the wall, however, there is severe probe interference, because the mass flow rate from the wall is some five times as large as that towards it. It is perhaps interesting that at this near-wall position the measured profiles (though seriously in error) are still sensibly the same for the smooth and rough channels.

The distribution of axial velocity U_3 (normalized by mean velocity $\overline{U}_3$) for

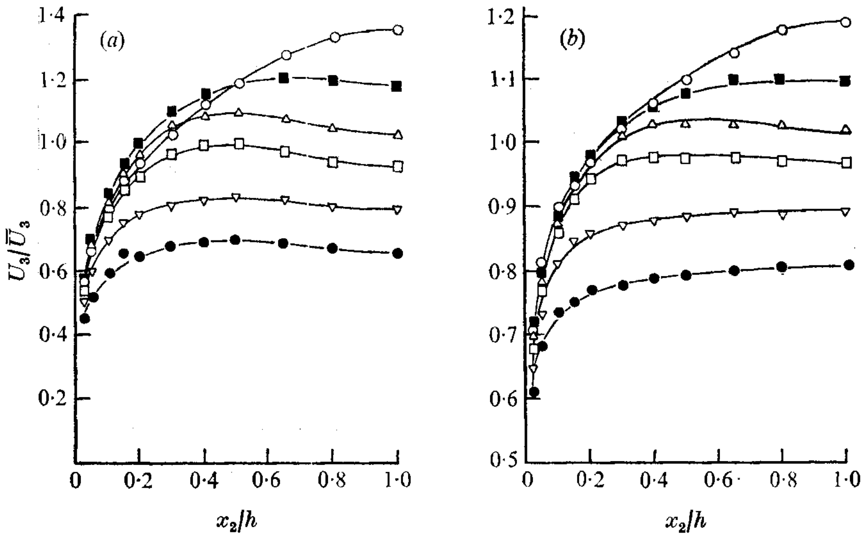

FIGURE 5. Axial velocity profiles in (a) rough and (b) smooth ducts; $Re = 215000$.

	●	▽	□	△	■	○
x_1/h	0·05	0·1	0·2	0·3	0·5	1·0

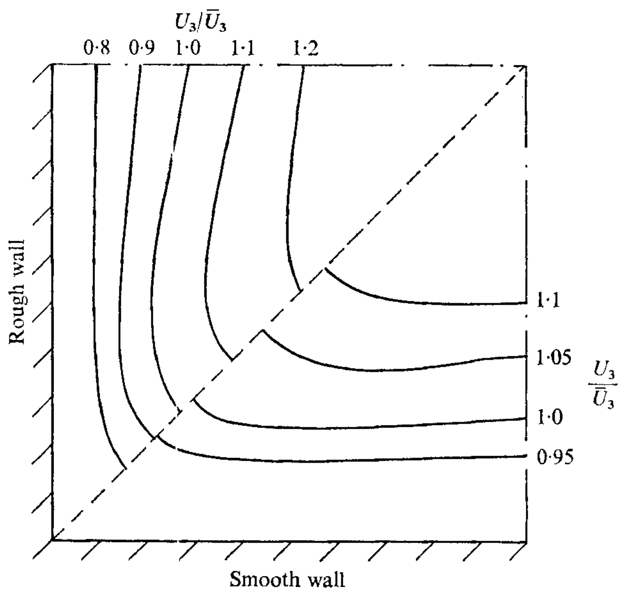

FIGURE 6. Axial velocity contours in smooth and rough ducts.

the two ducts is shown in profile form in figure 5 and by a contour plot in figure 6. These figures exhibit the accustomed bulges in axial velocity caused by the secondary motions; the effect is more marked for the profiles of flow in the rough channel. The result is expected since, as noted above, the ratio of secondary to primary velocity is greater in the rough than in the smooth duct. For the rough channel there was no discernible difference between either the axial- or secondary-flow profiles at the two Reynolds numbers investigated. The result indicates that the surface is 'fully rough'; this is consistent with the friction factor data for the channel (Launder & Ying 1971), which over a six fold range of velocity show no dependence on Reynolds number.

4. Conclusion

The main conclusion to emerge from this work is that the secondary velocity in a fully developed flow through a square-sectioned duct is sensibly independent of whether the duct is rough or smooth, provided that the velocity is normalized with the average friction velocity.

We wish to express our gratitude to the Berkeley Nuclear Laboratories of the C.E.G.B., who supported this research at Imperial College and also made available the test facility at the B.N.L. Mr H. G. Lyall and Mr C. J. Lawn, members of staff at the B.N.L., have been closely associated with the experimental work from the early planning through to completion; to them we express our sincere thanks.

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